An Investigation of The Relation Between 8th Grade Students’ Beliefs, Abstract Thought and Achievement; The Case of Mathematics

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ABSTRACT

The purpose of the current study was to examine the relationship between elementary 8th grade students’ mathematical beliefs, abstract thought and mathematical achievements in mathematics. This is a quantitative research using the relational survey model. The data collection tools utilized in the study were the “Mathematics Belief Scale” and the “Mathematical Abstract Thought Test”. The study group, formed through simple random sampling, was 518 8th grade students enrolled in a state school in Hendek, Sakarya, during the 2012-2013 academic year. The inferential statistical analyses conducted with the data yielded a positive relationship between students’ achievement scores and their mathematical beliefs and abstract thought levels, and between their abstract thought levels and mathematical beliefs. As a result, it was observed that the more positive students’ mathematical beliefs were, the more successful they were in mathematics, and the more positive their mathematical beliefs were, the higher their abstract thought levels were.

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Keywords: Mathematical Beliefs, Abstract Thought, Mathematical Achievement

Introduction

The rapid developments in information and technology necessitate individuals to keep up with these developments, to abandon simple logic and adopt a questioning and an internalizing mind in order to adopt effective ways of getting information easily. Mathematics education plays an important role in the occurrence of this information and the information society (Aydın, 2003). Mathematics is a highly important discipline as it has an impact on students’ future success in their (business) life. According to Asiala, et al. (1996) an individual can reach success only when concepts and structures are accurately internalized and applied in different situations within a rational framework. Moreover, the cognitive process, which is one of the profound stones of mathematics, will be completed by adopting the knowledge of computation (Üredi & Üredi, 2005). However, there are numerous factors that should not be ignored in order to complete the cognitive process successfully. Among these factors, the affective learnings have an essential line (Uysal Koğ, 2012), so both cognitive and affective domains of learning behaviours should be taken into consideration (Bacanlı, 2006).

Affective learning is not only a separate set of instructional objectives, but it is also used as a means to achieve cognitive learning (Berkant & Sürmeli, 2013). Learned helplessness conditions, self-confidence, interests, attitudes, beliefs, anxieties, values and such similar features are strong affective characteristics that enable teachers to understand their students comprehensively. Thus, being acquainted with the mentioned features of individuals will aid in understanding their condition and in predicting their future behaviours (Tekin, 1996). Furthermore, MEB (2009) states that students’ affective aspects may contribute to their
understanding of and liking for mathematics. Hence, an effective analysis of students’ cognitive and affective aspects throughout their educational life would enable educationalists to lead students to success.

Individuals unconsciously accumulate different affective experiences from birth throughout their lifetime. These affective accumulations may have the tendency to either motivate them towards success or prevent them from reaching success. The important point at this stage is considering the cognitive and the affective domains as not independent from each other. According to Demirtaş and Yağbasan (2004), there is a strong relationship between cognitive learning and the affective learning in education provided by schools. It is postulated that while affective entry characteristics account for 25% of the changes in learning outcomes, cognitive entry behaviours and affective entry characteristics account for 65% of the distribution in terms of levels of achievement. Accordingly, by changing students’ affective features to the positive, the difference between achievement rates can be reduced by 25% (Senemoğlu, 2001). In the light of this information, it can be concluded that to increase individuals’ levels of achievement, it is necessary to develop both the cognitive and affective characteristics of students.

Studies in the literature on cognitive and affective characteristics indicate that affective qualities have an impact on cognitive outcomes (Ağır, 2007; Altunçekiç, Yaman & Koray, 2005; Aksan & Sözer, 2007; Dündar, 2008; Hacıömeroğlu, 2011; Higgins, 1997; Kayan & Çakıroğlu, 2008; Mayer, 1998; Oğuztürk, Akça & Şahin, 2011; Pajares & Kranzler, 1985; Yaman & Yalçin, 2005; Yenice, 2012; Yılmaz, 2007). Although abstract thought is a high level cognitive ability, research topics taken up within this scope are generally limited to problem solving as a cognitive ability and such affective features as beliefs, learned helplessness, anxiety, and attitude. Not much emphasis is laid upon abstract thought although abstract thought has an important place in learning of mathematics since mathematics is a pure science (Yıldırım, 1988) and the acquired knowledge is also pure (Hassan ve Mitchelmore, 2006).

Abstract thought, which is one of the fundamental constructs this study is based on, can simply be defined as “the process of progressing from the concrete to the abstract” (Altun & Yılmaz, 2010: 313). Piaget and cognitive theorists who are Piaget’s advocates have maintained that abstraction is comprised of a series of mathematical processes and objects, and thus, with the establishment of a relationship between these objects based on their common features, students reach a higher level of mathematical object (Hershkowitz, Schwarz & Dreyfus, 2001). Those who support this approach have stated that abstraction is learned by examining samples during education and capturing the common points in them (Özmantar, 2004; Yeşildere & Türnüklü, 2008).

The role of abstract thought in reaching instructional objectives in mathematics is so crucial that it cannot be ignored. By means of abstract thought, problems requiring high level mathematical thinking can be solved more easily and the learning outcomes in these types of problems can be used in new problem contexts. King & Kitchener (1994), Kuhn (1991) and Perry (1968) assert that students’ mathematical beliefs underpin mathematical thoughts and reasoning ability, and those beliefs affect students’ cognitive performances. According to another point of view, students’ beliefs deeply impacted their cognition processes (Tang & Yu, 2008). Hence, there is a need for studies that investigate the relationship between abstract thought and mathematical beliefs. As Schoenfeld (1989) puts forward, mathematical belief is the unique mathematical world of an individual and one’s individual perspective towards mathematics. This being the case, the nature of the relationship between an individual’s perspective and level of abstract thought emerges as an important topic that needs to be studied.

To this end, the present study aimed to explore the relationship between mathematical belief, which is believed to be one of the strongest affective features in mathematics education, abstract thought, a high level cognitive ability, and mathematical achievement of 8th grade students. In addition, whether students’ mathematics achievement scores varied by their level of abstract thought (low, average, high) was also investigated. Furthermore, whether students’ level of abstract thought in mathematics, their beliefs and achievement levels varied with gender was also explored in the study.
Method

Research Model

The current study employed the relational survey model as it is aimed to reveal students’ conditions at one point in time and the relationship between their beliefs, their abstract thought levels, and their achievement scores at a certain period in time by means of a belief scale and an abstract thought test.

The Study Group

The study group was comprised of 518 (292 females and 226 males) 8th grade students, selected via simple random sampling, enrolled in a school in Hendek, Sakarya, during the 2012-2013 academic year.

The Data Collection Instruments

The data collection instruments conducted in the study was the Mathematics Belief Scale and the Mathematical Abstract Thought Test. In addition, an information form was administered to the students to determine their socio-demographical features and their mathematical achievement scores of 2012-2013 academic years first term.

The Mathematics Belief Scale

The Mathematics Belief Scale, developed by Steiner (2007) and adapted to the Turkish by Masal and Takunyacı (2012), was employed in the present study. The Mathematics Belief Scale is based on five dimensions: time, steps, understanding, usefulness and self-concept. The operational definition of these dimensions are as follows:

**Time:** The belief regarding the time needed to solve the maths problems

**Steps:** The belief that verbal problems cannot be solved in a simple way, that they can be solved by means of step-by-step procedures

**Understanding:** The belief that understanding concepts is important

**Usefulness:** The belief that mathematics is useful in daily life

**Self-concept:** The belief regarding the self-concept about mathematics.

Below there are some items about the five dimensions of The Mathematics Belief Scale:

- It should not take long time to understand mathematics (Time)
- It is necessary to use appropriate methods step by step to solve math problems (Steps)
- Finding the correct answer to a math problem is more important than what the answer is supposed to do (Understanding)
- Mathematics is beneficial and necessary (Usefulness)
- I have always succeeded in mathematics (Self-Concept).

Test-repeat test, Cronbach’s alpha, equivalent halves, and exploratory and confirmatory factor analyses were run to measure the reliability and validity of the Scale. In the scale developed by Steiner (2007) and adapted to the Turkish by Masal and Takunyacı (2012), the Cronbach’s alpha coefficient of the Mathematics Belief Scale was found to be .87, the equivalent halves reliability coefficient was .92 and the test-repeat test reliability coefficient was .83.

Mathematical Abstract Thought Test

The Mathematical Abstract Thought Test, developed by Uysal Koğ (2012), is comprised of a total of 18 questions in four cognitive areas, which are ‘Implementation’, ‘Analysis’, ‘Synthesis’ and ‘Evaluation’. The internal reliability indicator Cronbach’s alpha value of the test was found to be .94. Below, there are some questions related to abstract thought:
5. Two squares...

The difference between the measures of the perimeters of the 2 squares is 24 cm and the difference between the areas of the 2 squares 144 cm². What is the sum of the the measures of the perimeters of the 2 squares?

a) 96    b) 98    c) 100    d) 104

14. Complete the pattern

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

a) ***********  b) ***********  c) ***********  d) ***********

18. School bag….

When Tolgahan goes to school, he will take notebooks whose number is more than 4 and pencils whose number is less than 6. The notebooks and pencils that he will take with him are 9. How many pencils at most can he take with him?

a) 2    b) 3    c) 4    d) 5

The Test can be used for assessment in two different ways: First, the test with a total of 18 questions can be scored over 100 points to determine the abstract thought levels of students in mathematics. Secondly, the students’ answer sheets can be evaluated item by item to investigate the degree to which students use visualization in solving the maths questions. With the second method of assessment, the degree of impact of the activities used during visualization upon students’ level of abstract thought is sought.

In this study, the first way was used as a base to investigate the relationship between Abstract Thought Levels in Mathematics and Students’ Achievement Scores, and the other analyzed variables.

**Data Analyses**

The data were analysed by using the SPSS 18.0 program. The independent samples t-test was used in order to investigate whether students’ mathematical beliefs, Abstract Thought Levels in Mathematics and Students’ Achievement Scores indicate significant differences with respect to gender. Correlational analysis was also employed to investigate the relationship between the scores of beliefs, abstract thought and achievement. One-way analysis of variance (ANOVA) was conducted to examine whether Students’ Achievement Scores differ from each other according to their Abstract Thought Levels in Mathematics. Finally, the Tukey HSD (Honestly Significant Difference) test for multiple comparisons were employed to determine the direction of this difference. The statistical analyses were conducted at a significance level of .05. The first method of assessment in the Abstract Thought Test was employed. As a result of the Abstract Thought Test analyses, the standard deviation was calculated and one standard deviation above and below the mean was determined as average, high and low levels respectively.
Findings

The findings regarding the relationship between students’ mathematical beliefs, abstract thought levels in mathematics and achievement score are presented in Table 1.

<table>
<thead>
<tr>
<th>Variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Achievement Scores</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) Abstract Thought</td>
<td>.60**</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3) Mathematical Belief Total</td>
<td>.30**</td>
<td>.19**</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4) Self-Concept</td>
<td>.20**</td>
<td>.13**</td>
<td>.88**</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5) Usefulness</td>
<td>.27**</td>
<td>.15**</td>
<td>.70**</td>
<td>.40**</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6) Time</td>
<td>.19**</td>
<td>.11*</td>
<td>.47**</td>
<td>.33**</td>
<td>.09</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7) Steps</td>
<td>.13**</td>
<td>.05</td>
<td>.49**</td>
<td>.25**</td>
<td>.50**</td>
<td>.11*</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>(8) Understanding</td>
<td>.17**</td>
<td>.12**</td>
<td>.52**</td>
<td>.51**</td>
<td>.20**</td>
<td>.09</td>
<td>.17**</td>
<td>1</td>
</tr>
</tbody>
</table>

*p<.05  **p<.01

Table 1 indicates that there is a significantly moderate positive relationship between students’ abstract thought levels and achievement scores (r=.60, p<.01). Moreover, a significantly moderate positive relationship was found between students’ mathematical beliefs and achievement scores and a significantly weak positive relationship was found between achievement scores and the sub-dimensions of mathematical belief (usefulness, self-concept, time, understanding and steps) (p<.01).

The ANOVA test results indicating whether students’ mathematical achievement scores vary according to abstract thought levels (low, average, high) are presented in Table 2.

Table 2. The ANOVA test results of the students’ mathematics achievement scores in relation to their abstract thought levels

<table>
<thead>
<tr>
<th>Sums of Squares</th>
<th>sd</th>
<th>Mean Squares</th>
<th>f</th>
<th>p</th>
<th>Abstract Thought Level</th>
<th>X</th>
<th>Significant Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>56111,706</td>
<td>2</td>
<td>28055,853</td>
<td>96,589</td>
<td>.000</td>
<td>Low</td>
<td>47,35</td>
</tr>
<tr>
<td>Within Groups</td>
<td>152205,125</td>
<td>524</td>
<td>290,468</td>
<td>Average</td>
<td>56,89</td>
<td>L-H</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>208316,831</td>
<td>526</td>
<td></td>
<td>High</td>
<td>80,30</td>
<td>A-H</td>
<td></td>
</tr>
</tbody>
</table>

L: Low, A: Average, H: High Level

When Table 2 is examined, it is observed that achievement scores vary statistically across different levels of abstract thought (p<.05). As a result of the Tukey HSD analysis conducted to determine the direction of this variance, it was observed that the variance between students with a low abstract thought level and those with an average or high abstract thought level was in favour of the latter group. In addition, a difference was observed between students with average levels of abstract thought and those with high abstract thought levels, and the difference was in favour of students with high level abstract thought levels.

Findings regarding whether students’ mathematical beliefs vary across gender is presented in Table 3.
Table 3. The t-test results of mathematics belief and sub-dimension scores in relation to gender

<table>
<thead>
<tr>
<th></th>
<th>Gender</th>
<th>n</th>
<th>X</th>
<th>Ss</th>
<th>sd</th>
<th>t</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-Concept</td>
<td>Female</td>
<td>246</td>
<td>22.83</td>
<td>8.24</td>
<td>440</td>
<td>2.078</td>
<td>.038</td>
</tr>
<tr>
<td></td>
<td>Male</td>
<td>196</td>
<td>24.45</td>
<td>8.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Usefulness</td>
<td>Female</td>
<td>251</td>
<td>15.25</td>
<td>5.4</td>
<td>443</td>
<td>1.545</td>
<td>.123</td>
</tr>
<tr>
<td></td>
<td>Male</td>
<td>194</td>
<td>16.05</td>
<td>5.34</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time</td>
<td>Female</td>
<td>251</td>
<td>18.07</td>
<td>5.08</td>
<td>438</td>
<td>.571</td>
<td>.569</td>
</tr>
<tr>
<td></td>
<td>Male</td>
<td>189</td>
<td>17.80</td>
<td>4.91</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steps</td>
<td>Female</td>
<td>270</td>
<td>8.04</td>
<td>2.89</td>
<td>474</td>
<td>2.235</td>
<td>.026</td>
</tr>
<tr>
<td></td>
<td>Male</td>
<td>206</td>
<td>8.61</td>
<td>2.63</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Understanding</td>
<td>Female</td>
<td>276</td>
<td>11.37</td>
<td>3.44</td>
<td>480</td>
<td>4.407</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Male</td>
<td>206</td>
<td>12.83</td>
<td>3.80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mathematical Belief</td>
<td>Female</td>
<td>191</td>
<td>74.93</td>
<td>17.40</td>
<td>330</td>
<td>-1.904</td>
<td>.058</td>
</tr>
<tr>
<td></td>
<td>Male</td>
<td>141</td>
<td>78.48</td>
<td>16.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>Female</td>
<td>292</td>
<td>46.77</td>
<td>19.60</td>
<td>516</td>
<td>1.634</td>
<td>.103</td>
</tr>
<tr>
<td></td>
<td>Male</td>
<td>226</td>
<td>43.81</td>
<td>21.50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Abstract Thought</td>
<td>Female</td>
<td>292</td>
<td>61.12</td>
<td>19.31</td>
<td>516</td>
<td>1.529</td>
<td>.127</td>
</tr>
<tr>
<td></td>
<td>Male</td>
<td>226</td>
<td>58.42</td>
<td>20.69</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

When Table 3 is assessed, it can be stated that students’ mathematical beliefs do not vary significantly across gender \[t_{(330)} = -1.904, p>.05\]. However, in the self-concept, steps and understanding sub-dimensions of mathematical belief, a significant variance in favour of males can be observed.

According to the same table, it is observed that students’ mathematical abstract thought test scores \[t_{(516)} = 1.634, p>.05\] and their achievement scores \[t_{(516)} = 1.529, p>.05\] do not vary across gender.

**Discussion and Conclusion**

In the current study, the aim of which was to examine the relationship between students’ achievement scores, their abstract thought levels in mathematics and mathematical beliefs, a significantly positive relationship was found between achievement scores, abstract thought levels, beliefs, and all the sub-dimensions of mathematical belief (except between steps and abstract thought).

The Table 1 displays a significantly positive relationship between students’ mathematical beliefs and abstract thought scores in terms of both total scores and sub-dimension scores (except the sub-dimension of steps) \(p<.05\). It is a fact that students hold the common belief that mathematics is made up of abstract concepts and that comprehending abstract concepts is difficult (Moralı, Köroğlu & Çelik, 2004). In this study, a significantly positive relationship was found between students’ mathematical beliefs and abstract thought scores. Abstract thought refers to a cognitive concept involving higher-order, or complex, thoughts and, to be able to think in an abstract manner implies that one is able to draw conclusions or illustrate relationships among concepts in a manner beyond what is obvious (Apps, 2008). However, there are individual differences in this area. Progressing onto abstract processes is, apart from being a biological function, an environmental transmission; in other words, it is closely related to education (Atasoy & Ertürk, 2008). Within this context, the beliefs and attitudes that are formed with the influence of the environment and education can have a positive or negative effect. The relationship reported in the literature between abstract thought, inherent in the nature of mathematics, and positive mathematical beliefs supports the results of the present study.

The positive relationship between students’ mathematical beliefs and their achievement scores is consistent with the findings of some studies in literature (Handal, 2002; White et al., 2006). Furthermore, mathematical belief is to be aware of the skills to do a task and to believe that this task can be achieved (Chaplain, 2000; Zusho & Pintrich 2003). Since a positive mathematical belief will help an individual to overcome feelings that mathematics is difficult, it will have a positive impact on success. Therefore, individuals with higher levels of belief are more successful in solving problems (Blumenfeld, Soloway & Marx, 1991; Pajares & Miller, 1997; Radhawa et al., 1993). According to McLeod and McLeod (2002), beliefs not only support education but are also a major component of education. Consequently, that there is a
positive relationship between beliefs and achievement is not a surprising result. This is because it is a possible result that positive belief towards mathematics affect achievement in mathematics, and this positive belief has an important role in achievement in mathematics (Stylianides & Stylianides, 2014).

Another finding of the present study is that there is a significantly positive relationship between students’ achievement scores and their abstract thought levels. These findings are consistent with those derived from studies by Kincal and Yazgan (2010), Şaşmaz Ören and Tezcan (2008) and Uysal Koğ (2012). Thus, it can be said that all these findings related to the relationship between abstract thought and mathematical achievement stem from the nature of mathematics as mathematics is fundamentally abstract and rational in nature. According to all these results, it can be concluded that the higher the abstract thought levels of students, the higher their achievement levels in mathematics are. Thus, that individuals with a high level of abstract thought have high mathematical achievement scores is quite a common condition that helps students overcome problem solving process in terms of conceptual and procedural perspective.

One other finding the present study yielded was that students’ mathematical beliefs did not vary across gender. Furthermore, examination of the sub-dimensions of mathematical beliefs indicated no significant variation in the “usefulness” and “time” sub-dimensions, whereas a significant variation was detected in favour of males in the “self-concept”, “steps” and “understanding” sub-dimensions. In related literature, no significant variation was reported between students’ mathematical beliefs and gender by Akbaş and Çelikkaleli (2006), Aksan and Sözer (2007); Altunçekiç, Yaman and Koray (2005); Azar (2010); Duatepe Paksu (2008); Hacıömeroğlu and Şahin Taşkın (2010); Handal (2002); Pajares and Kranzler (1985); Yaman, Cansüngü Koray and Altunçekiç (2004). On the other hand, a significant variation in favour of males was found by Yenilmek & Kakmacı (2008). These conflicting results reported in literature may arise from the fact that the impact of gender on students’ beliefs cannot be completely explained. In the present study, the result that the sub-dimensions of the Mathematics Belief Scale was in favour of males can be accounted by males having a higher level of self-concept in the sub-dimensions of mathematical belief. In consistency with this finding, Yun Dai (2001) states that the variance emerging in the self-concept sub-dimension may have influenced both the ‘steps’ sub-dimension and the ‘understanding’ dimension positively in favour of males.

In the present study, it was found that students’ total scores in the Abstract thought test did not significantly vary across gender. While a study by Kincal and Yazgan (2010) reported no significant difference between the abstract thought test scores of male and female students, both Koray and Azar (2008) and Kılıç and Sağlam (2009) reported parallelism between students’ abstract thought ability and logical thought levels in favour of males and females, respectively. These contradictory findings can be said to derive from the different sampling groups included in the studies.

According to the results of the present study, it can be concluded that students’ gender did not affect their mathematical achievement significantly, indicating that students’ mathematical achievement is not related to their gender. This is a common result consistent with the findings of other studies in literature (Alkan Dilbaz, 2013; Bağçeci, Döş & Sarıca, 2011; Kılıç & Karadeniz, 2004).

In conclusion, mathematical belief, an affective feature, has an impact on abstract thought, a high-level cognitive ability. Hence, in order to increase students’ level of mathematical achievement, not only cognitive but also affective features should be taken into consideration. Taking both of these features into consideration is important since it can help teachers to understand students more comprehensively, and enable students to overcome the negative feelings that may arise in connection to mathematics, and in turn, reach success in mathematics.

References


